

FINAL EXAM SOLUTIONS:

16.

A $C_6^5 \Rightarrow 6C_5 = \frac{6!}{5!}$

$$\Rightarrow \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$$

answer = 6

B $C_5^5 \Rightarrow \frac{5!}{5!} \Rightarrow \frac{5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$

= 1

$5C_0 = 5C_5 = 1$

17 Multiple-choice exam that consists of nine questions

Binomial general formula

$$\sum_{k=0}^n C_n, k (p)^k (q)^{n-k}$$

$n=9$, correct answer = $\frac{1}{5}$, wrong answer

$q = \frac{4}{5}$

getting 1 correct

$$C_{9,1} = \left[\frac{1}{5}\right]^1 \left[\frac{4}{5}\right]^8 + C_{9,9} \left[\frac{1}{5}\right]^9 \left[\frac{4}{5}\right]^0$$

$$= 0.301989888 + 0.000000512$$

$$= 0.3019904 \approx 0.302$$

Probability of getting one correct / passing

$$= 0.302$$

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1, 2, 3, 13, 2, 7, 2, 6, 3, 2, 4

Arranging in ascending order

(a) 1, 2, 2, 2, 2, 3, 3, 4, 6, 7, 13

Median price is the one at the centre.

$$= \underline{\underline{3}}$$

(b) Mode price,

is the most appearing / high frequency price.

$$= \underline{\underline{2}}$$

(c) Sample mean

$$\frac{1 + 2 + 2 + 2 + 2 + 3 + 3 + 4 + 6 + 7 + 13}{11} = \frac{45}{11}$$

$$\approx 4.091$$

(d) sample standard deviation

$$\begin{aligned}
 s^2 &= \frac{1}{n-1} \sum (x - \bar{x})^2 = \frac{1}{11-1} \sum_{i=1}^{11} \left\{ (1-4.091)^2 + (2-4.091)^2 + \right. \\
 &\quad (2-4.091)^2 + (2-4.091)^2 + (2-4.091)^2 + \\
 &\quad (3-4.091)^2 + (3-4.091)^2 + (4-4.091)^2 + \\
 &\quad \left. (6-4.091)^2 + (7-4.091)^2 + (13-4.091)^2 \right\} \\
 &= \frac{1}{10} \left(9.554 + 4(4.372) + 2(1.190) + \right. \\
 &\quad \left. (0.00828 + 3.6443 + 79.3703 + 8.4623) \right) \\
 &= \frac{120.090718}{10} \\
 &= 12.0090718
 \end{aligned}$$

$$s^2 = 12.0090718 \Rightarrow s = \sqrt{12.0090718}$$

$$s = 3.47717$$

sample standard deviation ≈ 3.4772

19 Find $P(-1.03 < Z < 2.25)$ { from z-scores table }

$$P(Z < -1.03) = 0.1515, \text{ and } P(Z < 2.25) = 0.9878$$

$$P(-1.03 < Z < 2.25) = 0.9878 - 0.1515 = 0.8363$$

$$= 0.8363$$

20 Find $P(-1.20 < Z) = P(Z > -1.20)$

$$P(Z < -1.20) = 0.1151$$

$$P(Z > -1.20) = 1 - 0.1151 = 0.8849$$

$$= 0.8849$$

21 $\mu < 8,000$ pounds, $n=100$, $\bar{X}=6000$

$$\sigma = 200, \alpha = 5\% (0.05)$$

Null hypothesis: The mean of gold mine is greater or equal to 8000 pounds.

Alternative hypothesis: The mean number of gold mine is less than 8000 pounds.

Symbol

$$H_0: \mu \geq 8000$$

v/s

$$H_a: \mu < 8000$$

$$\alpha = 0.05$$

$$\text{Test statistics} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$$P\left(Z < \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}\right) = \frac{6000 - 8000}{\frac{200}{\sqrt{100}}}$$

$$\frac{-2000}{20} = -100$$

$$P(Z < -100) \text{ (from normal scores)}$$

$$p = 0.000$$

Since $p < 0.05$, thus we reject null hypothesis and conclude that the mean number of gold mine is less than 8000 pounds.

0.000 < 0.05, thus we reject null hypothesis and conclude that the mean number of gold mine is less than 8000 pounds.

22. claim $P \geq 75\% (0.75)$

$n = 1000$, 800 hand computers

$$\hat{p} = \frac{800}{1000} = 0.8 \quad p = 0.8$$

Null hypothesis: computers are less or equal to 75% of the households in the United States.

Alternative hypothesis: Computers are more than 75% of the households in United States.

$$H_0: P \leq 0.75$$

v/s

$$H_a: P > 0.75$$

$$\alpha = 0.05$$

$$\text{test-statistic} = Z_{\alpha/2} \sqrt{\frac{p'q'}{n}}$$

$$\hat{p} = \frac{800}{1000} = 0.8 \quad Z_{\alpha/2} = 1.96 \quad \sqrt{\frac{0.8 \times 0.2}{1000}} = 0.0126$$

(0.784) from normal table

$$P \geq 0.2177, P > 0.05, 0.2177 > 0.05$$

hence we fail to reject null hypothesis and conclude that computers are less or equal to 75% of the household in the united states.

$$23 \quad n = 144, \mu = 7 \text{ hrs}, S = 0.5$$

$$95\% = 1 - 0.05 = 0.95$$

$$Z_{\alpha/2} = 1.96$$

$$\bar{x} \pm Z_{\alpha/2} \frac{s}{\sqrt{n}}, \text{ but } \bar{x} = 7$$

$$\text{margin of error} = Z_{\alpha/2} \frac{s}{\sqrt{n}} \quad s = 0.5$$

$$Z_{\alpha/2} = 1.96$$

$$n = 144$$

$$M.E = 1.96 \left(\frac{0.5}{\sqrt{144}} \right) = 0.08166666$$

thus 95% confidence interval is as follows

$$(7 \pm 0.08166666), \text{ to 3 d.p.}$$

$$\text{low or bound} = 7 - 0.081666666 = 6.918333$$

$$\text{Upper bound} = 7 + 0.081666666 = 7.081666666$$

$$= \{ 6.918, 7.082 \}$$